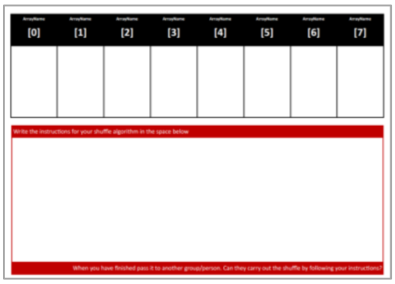
Investigating Shuffles

Teacher Notes to support Tenderfoot Unit 2: Clever Stuff For Common Problems – Going beyond simple algorithms

Devising an algorithm to shuffle cards is a lot easier than one to sort them. It provides a good introduction to manipulating values in an array. It also provides an introduction into investigating perfect shuffles, which leads to some magical connections between ideas in computer science.

Preparation required:
Shuffle Array Template + cards Ace to 8 in one suit for each pair / small group.
Perfect Shuffle Investigation sheet for each student.

A Shuffle Algorithm



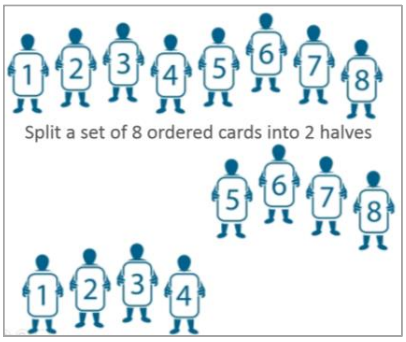
Children need repeated exposure to the same ideas in a variety of contexts before they sink in. This activity is designed to develop familiarity with manipulating arrays. It uses an array template with 8 positions and asks students to design an algorithm to shuffle cards. It works well as a small group or paired activity. Each group will need the cards Ace through to 8 in one particular suit.

There are many ways to shuffle cards, or other lists of items. There are no right answers, but the challenge is to articulate their method as an algorithm in the space below the array template. The aim is to think about the stages involved in a shuffle, articulate them in some form of pseudocode, specifying elements in the array by reference to their index position. Once they have articulated the steps involved, they can dry run their algorithm (or that of their neighbours) to test it.

If they are stuck for ideas, the presentation slides provide three possibilities. The first, repeatedly swapping random positions is the easiest to implement. Implementing the Knuth, or Fisher Yates shuffle is much more challenging. A couple of slides are included for you to use or study. They also provide an example for children of how to write their algorithm. Articulating (and coding) a Knuth shuffle is a good exercise for more able students and for staff learning about array manipulation. The key to understanding this shuffle is identifying the remaining list – or more specifically, the last position in the remaining list.

The third suggestion, a riffle shuffle is very challenging. Implementing a riffle shuffle is a good exercise for very able children. It also provides the basis for the investigation below.

Investigating Perfect Shuffles



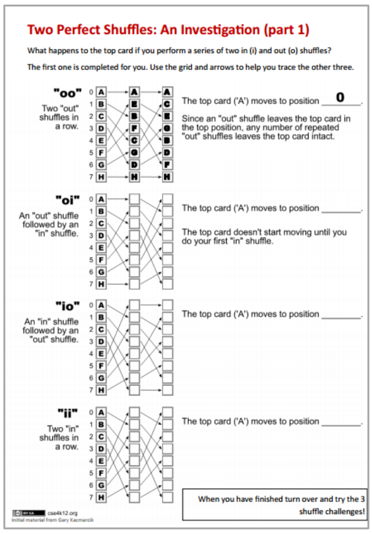
Most children will be familiar with how a riffle shuffle works. There are also some interesting properties associated with perfectly interleaved riffle shuffles, which they can explore. This is a pattern matching activity designed to reveal a surprising insight.

We can use the same array and 8 cards as before. It requires an explanation of an out-shuffle (leaving the top and bottom cards in place) and an in-shuffle, moving the top and bottom cards 'in'. The demonstration slides reveal the answer so children can check their result.

Once they understand, the first challenge is to perform 3 out-shuffles. There is a certain wonderment in finding the cards have returned to their original position.

With 3 in-shuffles the order hasn't returned, so encourage them to keep investigating. They should find it takes another 3 in-shuffles to return to the original order.

Controlling The First Card



We can move the first card (in array[0]) to array[1] with one in-shuffle. Challenge the students to try to move the first card to each subsequent position using a sequence of 'in' and 'out' shuffles. Start by trying to move it to array[2]. The answer is In-shuffle, followed by Out-shuffle. If some find a solution quickly, ask them to see if any other sequence of moves would achieve the same result.

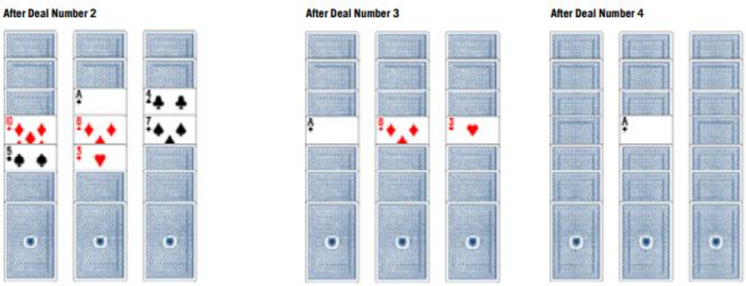
Combining two or more shuffles is an exercise best started in class but could continue for homework. The object is to familiarise children with array index positions whilst being able to trace cumulative changes. It isn't easy but the Perfect Shuffle Investigation gives a clear steer, with children only having to follow directions to derive the answer on the first side.

The second part of the investigation gets more complicated with less and less help available. Nonetheless, once completed, or even part way through, it should become apparent to the pupils that the pattern of In(I) Out(O) shuffles required to move to each index position is given by the binary number of the target e.g. to move to array[6] (binary 110) requires I I O. The exercise ends by asking for predictions. We are looking to develop the idea of pattern matching, and from that generalising so we can predict outcomes, based on what we have observed.

The 21 Card Trick

Moving cards around with a sequence of perfect shuffles is quite magical and the sort of manipulation involved in card forces. Unfortunately, perfect shuffles take a lot of practice! Paul Curzon and Pete McGowan have produced a series of magic books where each trick or illusion has explicit links to CS. The first volume focuses on card tricks. Thankfully, most require far less practice.

The 21 card trick is a classic card force, the connection with the perfect shuffle being easy to make. It can generate the sort of classroom 'wow' we wish to encourage amongst teachers. It involves dealing out 21 cards in a 3 by 7 matrix as shown right. Through 3 more successive deals, the chosen card is moved to the middle of the matrix. In performing this, we have an introduction, if needed to a 2 dimensional array and another possible algorithm to trace (or program) for very able students.



What we are trying to do here is reveal the links between seemingly disparate ideas in CS. Manipulating arrays makes a connection with binary. Unexpected patterns emerge that are surprising and stimulate a desire to delve deeper. Maybe students could experiment with a full set of cards?

If teachers wish to understand more about the maths involved, or indeed the link between perfect shuffles and clever methods to move data in memory, the lecture by Brent Morris (youtu.be/GV9W-vvhh-0) is a valuable use of 50 minutes.

The Magic of Computer Science is a card trick special. The 'self working' card tricks are all good examples of algorithms – following a set a defined steps to achieve the same result each time. They would make an excellent subject for a Hub meeting, with attendees volunteering to do a trick each. The magic book is included in the resources.