Analytical Models

Teacher Notes to support Tenderfoot Unit 10: Simulating Our World – Adventures in agent based modelling

A brief practical introduction to two analytical models. A short structured investigation to highlight some key ideas regarding computer models.

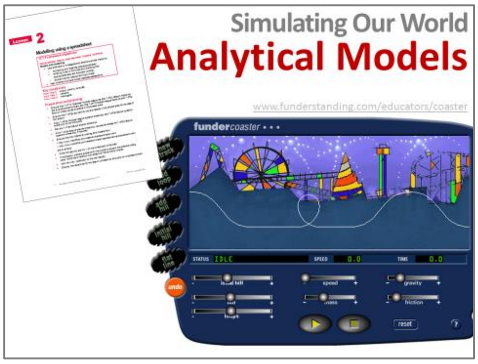
Preparation required:
Ensure Bridge Design software installed on all computers.

Computer Models

There are all sorts of models. Children from a very early age build them. It's claimed that Leonardo Da Vinci built models of flying machines to understand the flight of birds. Through model building people gain an understanding of the ideas embedded in them. Models help convey those ideas to others too. The advent of computers has made model making easier, allowing children to explore models and learn new ideas in the process.

There are many kinds of computer models. The old QCA Scheme of Work for ICT included units on models. One used a spreadsheet to model a Tuck Shop. The model was a mathematical model, its rules conveyed through formulae in cells. When something was sold, the stock went down, for example. We could call it an analytical model. Once built we could use it to answer 'what if' type questions. One problem with such models is that the degree to which students can develop them is limited by their mathematical knowledge. Even simple behaviour often requires maths that is beyond the average child at KS3.

That said, there are lots of engaging analytical models on the web for students to explore and get experience of trying to identify the underlying rules. A good starter activity for an introduction to modelling is exploring Funderstanding Roller Coaster: goo.gl/EyJn1w. This simulates some simple rules of physics. Funderstanding Rollercoaster is a deterministic model. By that, we mean the same settings will always generate the same behaviour. They therefore offer potential for exploring and trying to understand some of the underlying rules.



A Deterministic Model


Welcome to...

Bridge Designer 2016

Powered by Engineering Encounters

○ Create a New Bridge Design

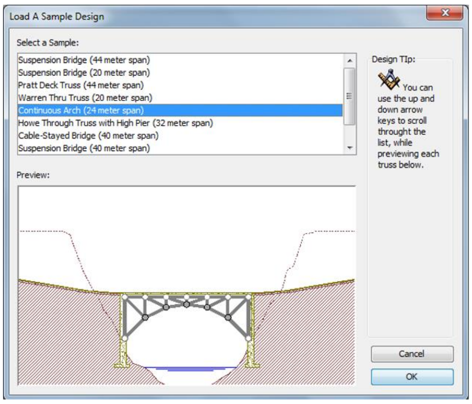
● Load a Sample Bridge Design.

○ Load an existing Bridge Design File.

Design tip:

When you use the Bridge Designer, your objective is to design a truss bridge that costs as little as possible, while still passing the load test with no member failures.

A very engaging deterministic model that has been around for many years is available from the Engineering Encounters website: goo.gl/4qa8p5. Formerly West Point Bridge Designer, it is the software used in the annual Bridge Design Contest, organised in the US. It is now an Open Source project, free to download, with a GNU license. Children can also join the main contest for fun, but prizes are only awarded to US students. Running a similar contest in your own school competition can be very motivational. The challenge is simple – to design the lowest cost bridge possible, capable of transporting a truck across the valley. Again, it is worth us spending ten minutes or so on the computers looking at this software.


Load A Sample Design

Select a Sample:

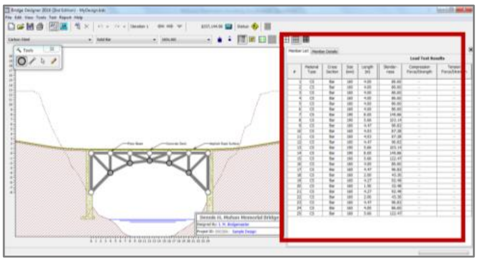
Suspension Bridge (44 meter span)
Suspension Bridge (20 meter span)
Pratt Deck Truss (44 meter span)
Warren Thru Truss (20 meter span)
Continuous Arch (24 meter span)
Howe Through Truss with High Pier (32 meter span)
Cable-Stayed Bridge (40 meter span)
Suspension Bridge (40 meter span)

Design TIp:

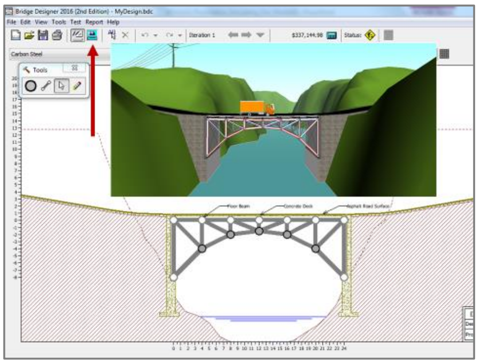
You can use the up and down arrow keys to scroll throught the list, while previewing each truss below.

Preview:

When Bridge Designer launches you are offered the option of creating a new design or loading a sample. In a contest, the ultimate goal is to design your own bridge, but students will learn most from exploring pre-built samples first. There are a range of different designs. Scrolling through gives an idea of the possible options open for you to investigate. To illustrate some of these let's all select a Continuous Arch with a 24m span as an example.



The bridge design is shown in the main left panel of the interface. Along the top are a range of menu options and toolbars. The right panel lists the details of each member (or beam) from which the bridge is constructed. There is a lot of detail here and it should be immediately apparent that there are a lot of underlying rules behind this model. For the moment let's close down the Member List – it can easily be opened from the View menu at a later stage.

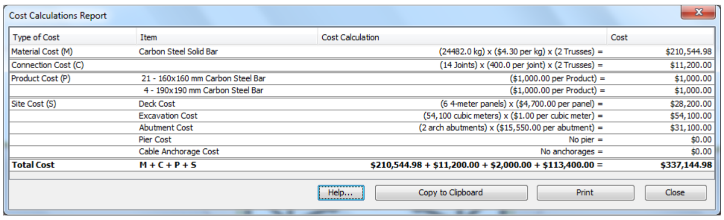


It is worth spending a moment familiarising yourselves with the interface. We have a completed bridge design but does it work? We can test it by running the Load Test animation (shown). As the truck drives over the bridge, the shades of red and blue indicate the load on the different members.

So that's the basic model, and the challenge is to shave the costs as far as possible whilst still allowing the truck to pass. The total cost is given in the toolbar, and selecting the calculator brings up a window giving a breakdown of those costs.

Systematic Investigation

In making sense of those costs we've just taken our first steps in decomposing the problem into smaller parts. We can now see the elements that make up the bridge and their proportionate costs.



Type of Cost	Item	Cost Calculation	Cost
Material Cost (M)	Carbon Steel Solid Bar	(24482.0 kg) x ($4.30 per kg) x (2 Trusses) =	$210,544.98
Connection Cost (C)		(14 Joints) x (400.0 per joint) x (2 Trusses) =	$11,200.00
Product Cost (P)	21 - 160x160 mm Carbon Steel Bar	($1,000.00 per Product) =	$1,000.00
	4 - 190x190 mm Carbon Steel Bar	($1,000.00 per Product) =	$1,000.00
Site Cost (S)	Deck Cost	(6 4-meter panels) x ($4,700.00 per panel) =	$28,200.00
	Excavation Cost	(54,100 cubic meters) x ($1.00 per cubic meter) =	$54,100.00
	Abutment Cost	(2 arch abutments) x ($15,550.00 per abutment) =	$31,100.00
	Pier Cost	No pier =	$0.00
	Cable Anchorage Cost	No anchorages =	$0.00
Total Cost	**M + C + P + S**	**$210,544.98 + $11,200.00 + $2,000.00 + $113,400.00 =**	**$337,144.98**

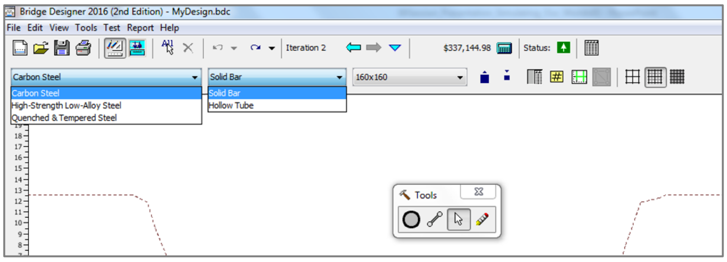
That alone allows children to begin to make more reasoned judgements when starting their own design. Excavation costs, for example are an early cost, determined by the height you decide to try to span the valley. Looking at the Cost Report how many variables can you identify?

We have the four main constituents; materials, connections, product and site costs. But each category is itself dependent on several variables. We can build high, or low, use arches, cables or trusses, include or leave out piers, use different materials for the road and the bridge and so on. With such complexity, it is impossible for children to get an understanding of how to optimise the overall model – which is why it makes for such a good competition. Children can come up with a wide variety of different strategies for bridge designs and will learn a lot from just trying different approaches.

How can we further our understanding of the bridge behaviour to develop a winning strategy? This is also a good discussion point for students. More generally, to add to a pedagogical framework, we want to move children from just using a model to exploring it. It is worth being explicit about this with children – getting them to share an understanding of how they can develop a toolkit for eventually building their own models.

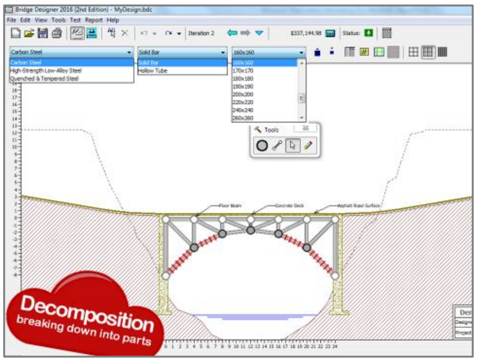
So how can we systematically explore the model? Children will often come to the idea intuitively that we need to isolate particular traits for observation whilst keeping other variables the same. We'll use an investigation of materials to illustrate that point. We can use our existing design. That way all other factors remain equal whilst we start to reason about the materials.

Use the Select Tool to highlight the first bridge member. Look at the materials drop down in the top left. You'll see we have a choice of Carbon Steel, High-Strength Low-Alloy Steel or Quenched & Tempered Steel. Which would be best? Given low price is our goal, the cheapest would seem the obvious solution.



With a member selected, we can investigate the relative cost of each material by observing the effect on total price. Which is cheapest? Carbon Steel. Matters aren’t quite that simple though. We also have a choice between solid bars and hollow tubes. Which is now the best option? Hollow tubes look like a cheaper option. How much can we save if we use tubes? Pupils can investigate. Selecting all the members (hold ctrl for multiple selections) and changing them to hollow tubes brings the price down to $187k.

Let’s now test the bridge! Some of the tubes aren’t strong enough and crumple! Clearly there is more to this than meets the eye. How might we now proceed? Students will come up with a variety of approaches and it’s worth encouraging a full discussion.



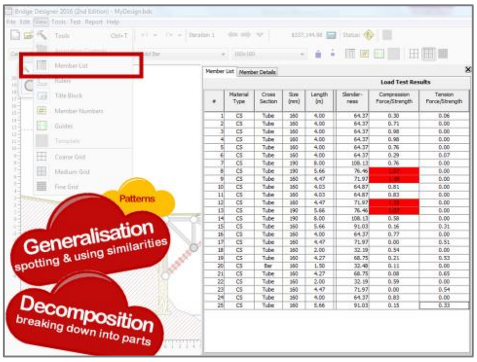
The design will indicate the members that have failed. Most children will realise there is a third ‘parameter’ we can alter – the thickness of the bar or tube. If we now increase the thickness of a tube, its’ price increases.

So how much do we increase it by? We now have a problem. How do we compare the relative performance of varying thicknesses of tubes and bars? It’s starting to get complicated!

Parameter Sweeping

Encourage the pupils to think about the goal. To get a thorough understanding of this model we need to run lots of tests with small increments in values. We need to find the cheapest tube or bar, of whatever material, at a thickness just above the point at which it will fail. This is known as Parameter Sweeping and it isn’t easy to do manually. We’ll return to this idea in a later session. Thorough parameter sweeping may not be practical but it may give pupils ideas about ways to ‘hone in’ their investigation.

We can delve further into the Load Test results by calling up the Member List. Although the list is intimidating, encourage students to see if they can spot a pattern between the members that have failed and those that are ok.



						Load Test Results	
#	Material Type	Cross Section	Size (mm)	Length (m)	Slender-ness	Compression Force/Strength	Tension Force/Strength
1	CS	Tube	160	4.00	64.37	0.30	0.06
2	CS	Tube	160	4.00	64.37	0.71	0.00
3	CS	Tube	160	4.00	64.37	0.98	0.00
4	CS	Tube	160	4.00	64.37	0.98	0.00
5	CS	Tube	160	4.00	64.37	0.76	0.00
6	CS	Tube	160	4.00	64.37	0.29	0.07
7	CS	Tube	190	8.00	108.13	0.76	0.00
8	CS	Tube	190	5.66	76.46	1.07	0.00
9	CS	Tube	160	4.47	71.97	[illegible]	0.00
10	CS	Tube	160	4.03	64.87	0.81	0.00
11	CS	Tube	160	4.03	64.87	0.83	0.00
12	CS	Tube	160	4.47	71.97	[illegible]	0.00
13	CS	Tube	190	5.66	76.46	1.07	0.00
14	CS	Tube	190	8.00	108.13	0.58	0.00
15	CS	Tube	160	5.66	91.03	0.16	0.31
16	CS	Tube	160	4.00	64.37	0.77	0.00
17	CS	Tube	160	4.47	71.97	0.00	0.51
18	CS	Tube	160	2.00	32.19	0.54	0.00
19	CS	Tube	160	4.27	68.75	0.21	0.53
20	CS	Bar	160	1.50	32.48	0.11	0.00
21	CS	Tube	160	4.27	68.75	0.08	0.65
22	CS	Tube	160	2.00	32.19	0.59	0.00
23	CS	Tube	160	4.47	71.97	0.00	0.54
24	CS	Tube	160	4.00	64.37	0.83	0.00
25	CS	Tube	160	5.66	91.03	0.15	0.33

This is another key concept in Computational Thinking, looking for patterns from which you can draw a general conclusion. Those that have failed have a Compression Force / Strength ratio greater than 1. Those that are fine are all <1. Armed with this knowledge, ask people to suggest a strategy for progressively improving their model.

Two key things could be drawn from the discussion. Some members will squash (compression) and others will stretch (tension). We need to get each member when under load as close to, but less than 1 in the Compression or Tension to Force ratio. We then need to compare the cost of different materials as close to their breaking point as possible. This might be the basis for a good group or class exercise. By selecting the Member Details tab you can get details of the degradation of the strength of a member as its length increases. This is a visual representation of parameter sweeping. The red marker indicates the strength required for that length, and you could incrementally increase the width of a member until it exceeded that level. We could go on, and on – this is just one parameter that determines the aggregate behaviour of the model. The more you delve into Bridge Designer the deeper your analysis can go, but unless you restrict the options for children to investigate, they learn little about the underlying rules.

In summary analytical models have value for giving pupils opportunities to use them to investigate problems. With care they may offer some possibilities for exploring underlying rules, but this often depends on restricting the exploration to one or two parameters. Because analytical models rely on writing rules to determine aggregate behaviour, the maths involved, beyond really trivial examples is usually beyond students at KS3.